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Rupture of Rubber. X. The Change in Stored Energy on Making a Small Cut in a Test Piece Held in Simple Extension

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INTRODUCTION

Rivlin and Thomas[1] have shown that when a small cut or tear is made in one edge of a test piece stretched in simple extension the change in the total stored energy in the test piece is given by:

$$W_t - W = K(\lambda)c^2hE \tag{1}$$

where W_t and W are the total stored energies before and after the cut is made, c and h are, respectively, the length of the cut and the thickness of the test piece measured in the undeformed state, E is the stored-energy density appropriate to the extension ratio λ in the simple extension region, and $K(\lambda)$ is a numerical factor that varies with λ. The derivation assumes that (*1*) the grip separation l is maintained constant, (*2*) c is large compared with the radius of curvature of the tip of the cut or tear, (*3*) c is small compared with the width of the test piece, and (*4*) the cut is sufficiently far from the grips to affect only the central region of simple extension. Equation (1) is the equivalent, in terms of strain energy, of an expression for the stress concentration produced by the cut or tear. In the case of highly extensible materials it forms the sole expression for such effects, as the calculation of the stress-strain distribution is intractable. Differentiation of eq. (1) with respect to c at constant l gives immediately the energy supply for tearing from which, by application of the energy criterion for tearing, the effect of cut length and strain on the tear behavior of the specimen can be ascertained.[1]

The problem of calculating the factor $K(\lambda)$ is intractable for large strains, and hitherto K has been evaluated, and eq. (1) tested, by comparing tear measurements made on this type of test piece with corresponding measurements made on other types of tear test piece for which the energy supply for tearing can be calculated explicitly.[1] These comparisons indicated that K has a value of about 2. However, recent work on rupture and fatigue failure in tensile specimens[2,3] indicates that it would be desirable to obtain a more accurate evaluation. A method involving only the elastic properties of the rubber should be inherently more accurate than one involving the rupture properties, and experiments that have been made to evaluate K in this way, for extensions from 5 to 200%, are described in the present paper. These experiments also provide a more stringent test of the accuracy of eq. (1) than has been possible hitherto.

EXPERIMENTAL

Method of Measurement

If the force on the test piece, as shown in Figure 1(*a*), is F_i in the absence of a cut and is F in the presence of a cut of length c, then $W_i - W$ is given as:

$$W_i - W = \int_{l_0}^{l} (F_i - F)\, dl \tag{2}$$

and, from eq. (1), $K(\lambda)$ is given as:

$$K(\lambda) = \int_{l_0}^{l} [(F_i - F)/c^2h]dl/\int_1^{\lambda} (F_i/A_0)\, d\lambda \tag{3}$$

where l_0 is the grip separation in the undeformed state and A_0 is the undeformed cross-sectional area of the test piece. If $l/l_0 \simeq \lambda$ (λ being the

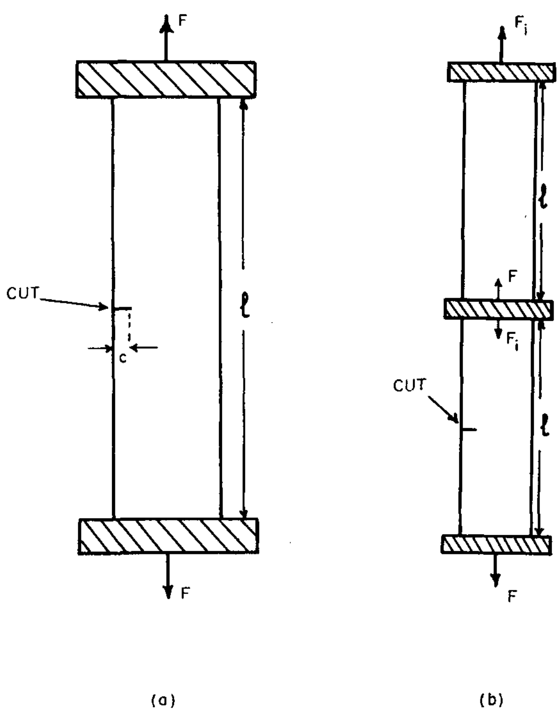

Fig. 1. (*a*) Simple extension test piece with a small cut in one edge. (*b*) Test piece with center grip.

extension ratio in the central region of simple extension), then eqs. (2) and (3) reduce to:

$$W_t - W = \int_1^\lambda l_0(F_t - F)\, d\lambda \tag{4}$$

and

$$K(\lambda) = \int_1^\lambda [l_0(F_t - F)/c^2h]\, d\lambda/\int_1^\lambda (F_t/A_0)\, d\lambda \tag{5}$$

It will be seen from these relations that $K(\lambda)$ can be evaluated from measurements of the forces F_t and F for various values of l/l_0. The experimental difficulty arises from the fact that, if the conditions assumed in the derivation of eq. (1) are satisfied $(F_t - F)$ is small compared with F_t and, according to the particular experimental procedure adopted, may be obscured by variations in F_t (between test pieces, from repeated stretching, or from stress relaxation). Variation of elastic properties between test pieces is difficult to control, and it seems preferable to measure $(F_t - F)$ on the same test piece. The remaining effects may then be considerably reduced by adopting the experimental arrangement shown in Figure 1(*b*). Measurement of the total force F_t is made at either of the outer grips, and $(F_t - F)$ is obtained from measurement of the out-of-balance force on the middle grip when a cut is made in the operative portion of the test piece (the lower half, say).

The procedure with the above arrangement can be either (*1*) to stretch the two halves of the test piece by equal amounts, first without a cut and then with one, the forces on the middle and outer grips being measured at appropriate intervals, or (*2*) to stretch the two halves of the test piece by equal amounts to some predetermined extension, the force on the middle grip then being measured for various lengths of cut. Procedure (2) entails measurements on several test pieces to obtain $(F_t - F)$ as a function of l/l_0 but it is probably the more sensitive method with low extensions. It was the one adopted for the present experiments. Procedure (1) may be the more suitable at high extensions.

Experimental Arrangement and Procedure

The test piece was mounted vertically in a test frame as shown in Figure 2. Spacers attached to the frame located the outer grips in parallel disposition at a fixed separation of about 15 cm. The tension in the test piece was sufficient to maintain the grips in position. The middle grip was mounted on a spring-hinged pantograph which kept it parallel to the outer grips but allowed free movement in the vertical direction. A scale pan was attached to the lower end of the middle grip mounting, and the weight of the whole assembly was counterbalanced by a helical spring. The vertical movement of the middle grip mounting was restricted to a few tenths of a millimeter by the fixed stop shown, and contact with this stop on downward movements was indicated by a neon lamp. In this position of contact with the stop, the middle grip was equidistant from the outer grips. As the cut was always made in the lower half of the test piece, the out-of-balance force on the middle grip was in the upwards direction, and it was

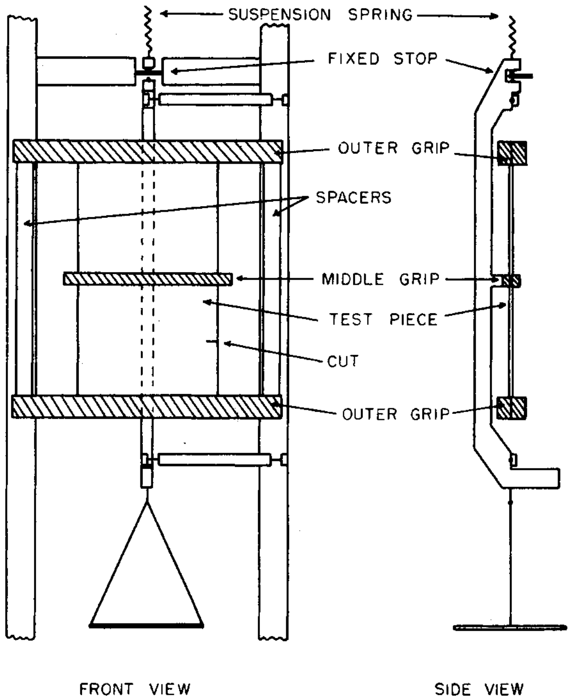

Fig. 2. Experimental arrangement.

counterbalanced by weights added to the scale pan, to maintain the contact against the stop. This arrangement gave a sensitivity of about 0.1 g. in measurement of the out-of-balance force for a total force on the test piece of the order of 1 kg.

The outer grips were attached to the test piece after it had been stretched out in a jig. Spacers of the same length as those in the test frame were disposed on either side of the stretched test piece and were used to locate the grips while they were being clamped in position. The test pieces were made sufficiently long that, when stretched, there was a central region rather more than 15 cm. in length that was substantially in simple extension. Prior to the placing of the grips in position, the uniformity of extension across the width of the test piece in this central region was checked by measurement of the separation of bench marks that had been placed on the test piece in the undeformed state. After the grips had been clamped in position and the outer portions of stretched rubber had been cut away, the test piece was transferred to the test frame, and the middle grip was then clamped in position.

The above procedure facilitated correct positioning and even clamping of the grips and reduced to negligible proportions the regions of complex deformation adjacent to the grips. The latter point was checked by comparing measurements of l/l_0, the ratio of the middle and lower grip separations in the deformed and undeformed states, with measurements of the extension ratio λ in the central region well removed from the grips. It was found that l/l_0 was equal to λ within the experimental accuracy, and the simplified eqs. (4) and (5) could therefore be used. The reduction to negligible proportions of the regions of complex deformation adjacent to the grips also enabled relatively wide and short test pieces to be used, with advantages in sensitivity as may be seen from eq. (4).

After the test piece had been mounted in the test frame the force on the middle grip was measured at intervals over a period of about 15 min. The initial force was normally not more than 5–10 g., and did not vary by more than a few tenths of a gram over this period. The force F_t on the lower grip was then measured by hanging weights from the grip until it hung just clear of the spacers. The weights were then removed and measurement of the force on the middle grip resumed. The measurement of F_t usually disturbed this reading slightly, but after a further period of about 15 min. the readings were constant to about a tenth of a gram. A small razor cut was then made in the middle of one edge of the lower half of the test piece. The tip of the cut was lightly smeared with ink, and after this had dried the force on the middle grip was measured. This procedure was repeated for successive increments in the length of the razor cut. These readings, after subtraction of the initial reading for no cut, gave the change in force, $(F_t - F)$, for the various cut lengths c. The time interval between the force readings was about 3 min. Subsidiary tests showed that the force reading in the presence of a cut remained substantially constant for periods comparable with the total time of the measurements.

The remaining measurements were made after the test piece had been removed from the test frame and allowed to relax overnight. The width and thickness of the test piece in the vicinity of the cut were measured, and also the distance between the marks left by the middle and lower grips, this being taken as the undeformed length, l_0. The test piece was then cut in two at the position of the razor cut and the cut lengths c measured with a travelling microscope. The successive positions of the tip of the razor cut were readily distinguishable with the aid of the ink marks that had been made. The measurements were made on both portions of the test piece and the readings were averaged.

The above procedure was repeated for various extension ratios, a fresh test piece being used in each case. A total cut length of at least 1–2 mm. was necessary to obtain adequate force and cut length data, and this requirement, together with the tear resistance of the rubber, set the limit on the maximum extension ratio that could be employed. For all but the softest of the vulcanizates, the maximum extension ratio was well below that for incipient crystallinity. The minimum extension ratio employed was about 1.05.

The test pieces were cut out with a template from sheet rubber. Sheets about 1 mm. thick were normally used. The test pieces were cut out to various lengths from 20 to 30 cm., depending on the extension ratio to be used, and to a width of 5 cm. for extension ratios greater than 1.3. For lower extension ratios the width was increased to 9 cm. so that relatively long razor cuts (ca. 1 cm.) could be made.

Materials

Gum vulcanizates of natural rubber were used for the experiments, as these materials have good elastic and tear resistance properties. The vulcanizing recipes and cures of the four vulcanizates used are given in Table I.

TABLE I
Vulcanizing Recipes and Cures

	A	B	C	D
Mix (parts by weight)				
Rubber	100	100	100	100
Dicumyl peroxide	—	—	3	—
Sulfur	1.5	3	—	3
Santocure	—	—	—	1
MBT[a]	0.3	0.5	—	—
Zinc oxide	5	5	—	5
Stearic acid	0.5	1	—	0.75
Antioxidant	1	1	—	1
Curing temperature, °C.				
	140	140	140	126
Curing time, min.				
	45	45	50	90

[a] Mercaptobenzothiazole.

Stress–strain measurements in simple extension were made according to the procedure described by Mullins[4] on dumbbells cut from the vulcanized sheets, and the elastic constants C_1 and C_2 of the following relation were derived:

$$f/2A_0 = (\lambda - 1/\lambda^2)(C_1 + C_2/\lambda)$$

where f is the load and A_0 the undeformed cross-sectional area of the specimen. The values obtained are given in Table II.

TABLE II
Elastic Constants of the Vulcanizates

	A	B	C	D
C_1, kg./cm.^{2}	0.71	1.22	1.61	2.44
C_2, kg./cm.^{2}	0.82	1.00	0.74	1.14

RESULTS AND DISCUSSION

Typical results for the change in force $(F_t - F)$, with cut length c are given in Figure 3. The particular results shown were obtained with the

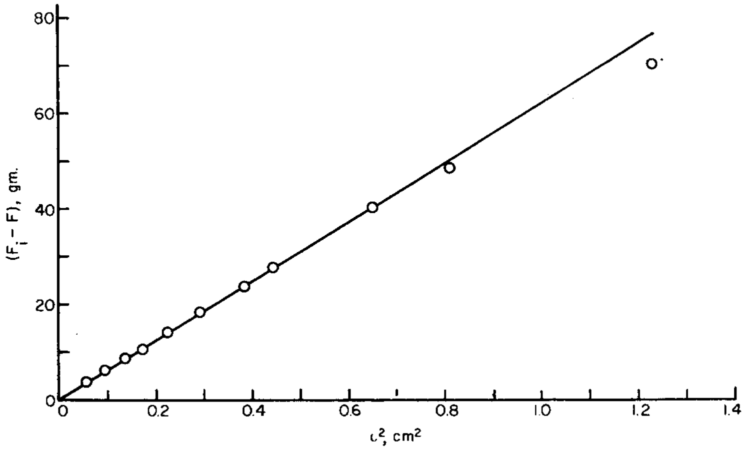

Fig. 3. Typical $(F_i - F)$ vs. c^2 plot: results for vulcanizate A at extension ratio of 1.25; test piece thickness approximately 0.1 cm.

vulcanizate A; the test piece width and thickness were 9 and 0.1 cm., respectively, and the extension ratio was 1.25. It will be seen that $(F_i - F)$ is accurately proportional to c^2 for small values of c, in conformity with eqs. (1) and (4). Measurements made at an approximately constant extension ratio with test pieces of 9 and 5 cm. width, and also with test pieces whose undeformed lengths l_0 differed by some 50%, indicated that the slope of the initial linear portion of the $(F_i - F)$ vs. c^2 plots was unaffected by the change in test piece width and was inversely proportional to l_0, as required by eq. (4). The results also indicated that $(F_i - F)$ was proportional to c^2 within the experimental accuracy for values of c up to about one fifth of the width or undeformed length of the test piece. Presumably, the departures from proportionality at larger values of c are associated with the reduction in the cross-sectional area of the test piece in the region of the cut and with interference of the grips with the relaxation of the material around the cut. Further experiments, in which test pieces with thicknesses of 0.05, 0.1, and 0.15 cm. were used, showed that the initial slope of the $(F_i - F)$ vs. c^2 plots was proportional to the test piece thickness, in conformity, again, with eqs. (1) and (4).

Values of $(F_i - F)/c^2$ for the various extension ratios employed were obtained from the initial slope of the $(F_i - F)$ vs. c^2 plots, and $l_0(F_i - F)/c^2h$ was then calculated, h being the thickness of the test piece measured in the vicinity of the cut. Values of $l_0(F_i - F)/c^2h$ for the vulcanizate D are shown plotted against the extension ratio λ in Figure 4, together with values of total stress F_i/A_0 against λ. The curve drawn through the F_i/A_0 vs. λ data is that given by the stress–strain data from which the elastic constants C_1 and C_2 were obtained. The agreement between the curve and the plotted points shows that the differences in the elastic properties of the various test pieces were small, and this was found to be true also of the

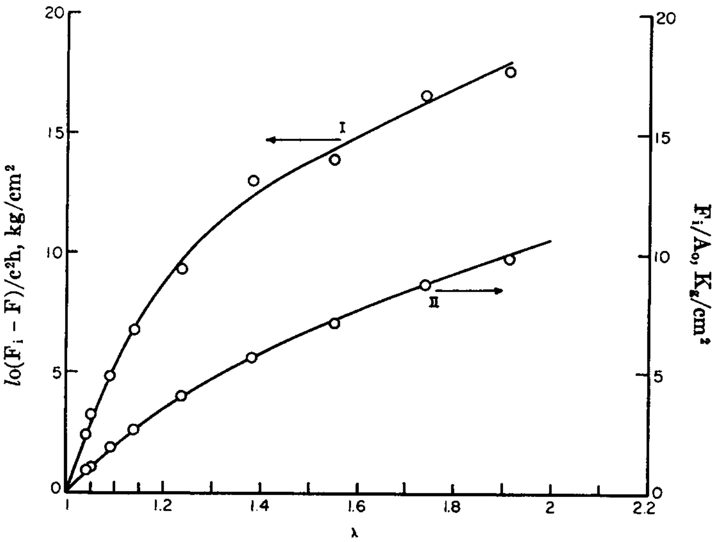

Fig. 4. Plots of $l_0(F_i - F)/c^2h$ vs. λ (I) and F_i/A_0 vs. λ (II) for vulcanizate D.

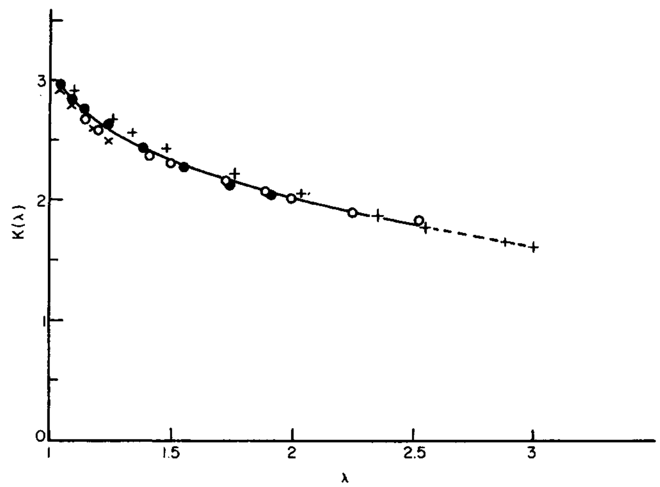

Fig. 5. Values of $K(\lambda)$ plotted against extension ratio. Results for: (+) vulcanizate A; (O) vulcanizate B; (×) vulcanizate C; (●) vulcanizate D.

other vulcanizates. As will be seen from eq. (5), $K(\lambda)$ is given by the ratio of the areas under the $l_0(F_i - F)/c^2h$ vs. λ and F_i/A_0 vs. λ curves. The areas were obtained by graphical integration and, from these, the values of K at the various extension ratios were derived. The values of K obtained with the different vulcanizates are shown plotted against λ in Figure 5. They show satisfactory agreement.

The results of Figure 5 show that K decreases from a value of about 3 at small extensions to a value somewhat below 2 at 200% extension, and this confirms the previous rough estimate from tear data.[1] An estimate of the theoretical value of K for small strains can be obtained from the classic solution for the related problem of a crack in the middle of a sheet (stretched in a direction perpendicular to the crack). In this case the decrease in the total stored energy of the sheet for a crack of length $2c$ is $2\pi c^2hE$ (see, for example, Timoshenko[5]). This suggests that the corresponding numerical factor, i.e., K, for an edge crack should be close to π. It will not be exactly π, as the plane of symmetry of the sheet (the plane perpendicular to the crack) is not strictly force-free and does not therefore correspond precisely to the free edge in the edge crack problem. The above experimental value for K at small extensions is in fair agreement with the theoretical estimate. It may be noted that in the experimental evaluation of K the contribution of a surface free-energy term to the changes in the potential energy of the system has been neglected. This contribution becomes important, however, only at very low stored-energy densities and small cut lengths and is negligible under the conditions that prevailed in the experiments.

As will be seen from Table II, the vulcanizates varied widely both in small-extension modulus (given by $C_1 + C_2$), and in the shape of the load–extension curve at higher extensions (governed by C_1/C_2). The results of Figure 5, which indicate that the value of K does not vary appreciably with the vulcanizate, therefore confirm eq. (1) in respect to the dependence of the change in total stored energy ($W_i - W$), on the stored-energy density E. As has been previously indicated, the experiments also confirm eq. (1) in respect to the dependence of ($W_i - W$) on cut length c and test piece thickness h. The experiments thus establish the accuracy of eq. (1) and provide data for its numerical evaluation, enabling the energy criterion for tearing[1] to be applied quantitatively in cases in which a cut or crack is present in a rubber specimen strained in simple extension.

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Synopsis

Application of the tearing energy criterion (Part I of this work) to the problem of the tearing of a rubber strip that contains a small cut or tear of length c in one edge and is stretched in simple extension is based on the relation $W_i - W = K(\lambda)c^2hE$, where W_i and W are the total stored energies of the strip in the absence and presence of the cut, respectively, h is the thickness λ^0 of the strip, E is the stored-energy density in the central region of simple extension, and $K(\lambda)$ is a numerical factor that varies with the extension ratio λ in this region. An experimental method is described for measuring $(W_i - W)$, which enables the factor $K(\lambda)$ to be evaluated. The results indicate that $K(\lambda)$ decreases from a value of about 3 at low extensions to a value somewhat below 2 at $\lambda = 3$. The experiments also provide a more stringent test of the accuracy of the above relation than has been possible hitherto.

Résumé

L'application du critère de l'énergie de rupture (partie I) au problème du déchirement d'une lamelle de caoutchouc qui contient d'un côté une petite incision ou déchirure de longueur c et qui est étirée en extension simple, se base sur la relation: $W_i = W = K(\lambda)c^2hE$; W_i et W sont les énergies totales accumulées de la lamelle, respectivement en absence et en présence de l'incision, h en est l'épaisseur, E est la densité le l'énergie accumulée dans la région centrale de l'extension simple, et $K(\lambda)$ est un facteur numérique qui varie avec l'extension moyenne λ dans cette région. Une méthode expérimentale a été décrite afin de mesurer $(W_i - W)$, ce qui nous permet d'évaluer le facteur $K(\lambda)$. Les résultats indiquent que K(λ) diminue d'une valeur d'environ 3 aux petites extensions à une valeur un peu moins que 2 à $\lambda = 3$. Les expériences pourvoient aussi un test plus rigide afin de controler l'exactitude de la relation citée ci-dessus.

Zusammenfassung

Die Anwendung des Kriteriums für die Reissenergie aus Teil 1 auf das Reissen eines Kautschukstreifens, der einen kleinen Schnitt oder Riss von der Länge c in einer Kante enthält und durch einfache Dehnung gestreckt wird, beruht auf der Beziehung: $W_i - W = K(\lambda)c^2hE$, wo W_i und W die gesamte, im Streifen in Ab- und Anwesenheit des Schnittes gespeicherte Energie, h die Dicke des Streifens, E die gespeicherte Energiedichte im zentralen Bereich einfacher Dehnung und $K(\lambda)$ einen numerischen, von Dehungsverhältnis λ in diesem Bereich abhängigen Faktor bedeutet. Eine experimentelle Methode zur Messung von $(W_i - W)$ wird beschrieben; es wird so die Ermittlung des Faktors $K(\lambda)$ ermöglicht. Die Ergebnisse zeigen, dass $K(\lambda)$ von einem Wert von etwa 3 bei niedriger Dehnung auf einen Wert etwas unterhalb 2 bei $\lambda = 3$ abnimmt. Die Versuche liefern einen strengeren Beweis für die Genauigkeit der oben angegebenen Beziehung als es bisher möglich war.

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